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Mathematical Picture of the World in Philosophy

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Abstract: *This article discusses the basic information about the creation of a mathematical picture of the world.*

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Introduction: The mathematical picture of the world as a form of systematization of mathematical knowledge differs from mathematical theory. If the mathematical picture of the world describes an object, distracting from the process of obtaining knowledge, then mathematical theory contains logical means of both systematization of knowledge about the object and verification of their truth. The mathematical picture of the world is based on a certain set of initial fundamental mathematical concepts, hence the dominant position of a particular branch of mathematics. As science develops, the leading group of concepts gives way to other concepts, terms. The mathematical picture of the world cannot be found ready-made in any sources and considered as an independent object of study, that is, its analysis causes considerable difficulty. The change in the mathematical picture of the world occurs as a result of revolutionary transformations in science. There are discoveries in mathematics that entail changes in the philosophy of mathematics, in the understanding of its subject, methods, and connections with other sciences. In mathematics , such events include the following:

- The emergence of the very idea of mathematics as a deductive science;
- Discovery of incommensurable quantities;
- Discovery of infinitesimal analysis;
- Discovery of non-Euclidean geometry;
- Computerization of knowledge.

Methods: In ancient Egypt and Babylonia, the entire cognitive process took place through calculations, while the instrument of counting was the fingers of a person, i.e. counting on the fingers. The subject of practical mathematics of the first level is measuring and comparative actions mediated by learning, in the process of which new tasks appear that do not follow from practice. Thus, within the framework of practical mathematics of the first level, there is a gradual internal stratification of types of problems with a focus on general methods for solving typical problems. The mathematics of Ancient Egypt and Mesopotamia consisted of two components: geometric and arithmetic knowledge. Geometric knowledge arose in the process of agricultural practice, and arithmetic knowledge - from the recalculation of objects, exchange relations. At the first stage of practical mathematics, a group of terms of mathematical objects and a group of terms of operations are formed. For objects that differ in size and numerical coefficients, it is found that the concepts of operations found are equally applicable. Gradually, the principles of solving such problems are identified on the basis of the similarity of geometric shapes and the similarity of numerical calculations. There are key terms necessary for the formation of elements of general methods. For geometric problems, such a term is "similar", and for arithmetic problems - the term "proportional". At the first stage of the development of mathematics, mathematical problems were solved arithmetically. Geometry did not stand out as an independent field of knowledge. The classification of tasks was carried out not by methods, but by their applied topics.

In mathematical texts, key terms and terms of logical relations penetrate in Ancient Greece in the VI century BC, which meant the completion of a qualitative leap in the development of the terminological apparatus and the transition of mathematics to a theoretical method of systematization of mathematical knowledge. The emergence of terms of logical relations opens up the possibility of knowledge growth and the emergence of new mathematical objects and operations. In ancient Greece, mathematics was first developed as an independent branch of knowledge. Mathematics has turned from a semi-empirical science into a deductive science. The prescription presentation was replaced by a system of logical proofs. For the first time, axiom systems were formulated, on the basis of which the majestic buildings of geometry and number theory were built.

The mathematical picture of the world grew out of the philosophy of Pythagoras and Plato. The mathematical picture of the world is based on the idea of the Cosmos as an ordered expression of initial entities, for example, for Pythagoras these were numbers. The central core of the mathematical picture of the world was considered arithmetic, which was interpreted in early Pythagoreanism as the central core of the entire Cosmos, and geometric problems - as problems of arithmetic of integers, rational numbers, geometric quantities - as commensurate. The mathematical picture of the world is harmonious: extended bodies are subject to geometry, celestial bodies to arithmetic. Plato separated the world of things from the world of ideas. According to Plato, arithmetic is considered an ideal science, since numbers form ideal objects, and geometry is only half perfect, because it is located between the world of ideas and the world of things.

The discovery of the incommensurability of the side of a square and its diagonal dealt a serious blow to ancient mathematics. Mathematicians began to think about the foundations of their science. They chose geometry as the basis of mathematics, which managed to represent relations inexpressible with the help of arithmetic numbers and relations. Evdox Knidsky developed the theory of proportions and its applications to geometry; the method of exhaustion is the study of complex forms of incommensurability by means of unlimited reduction of residues.

Euclid's geometry largely determined the structure of the whole science, where the initial concepts are set by a system of axioms. The main drawback of Euclid's "Principles" is the lack of concepts of continuity and functional dependence. Geometric algebra began to slow down the development of ancient mathematics. There was no place for negative, imaginary numbers in this algebra. Therefore, it was necessary to abandon the geometric language and look for a new, more general and flexible language. Such a language was found in the first centuries of the new era - in the era of Hellenism.

In the works of scientists of the "Greek Renaissance" there was a tendency to turn to computational mathematics, to expand the concept of number, to abandon geometric algebra. Mathematics in the countries of the East was the doctrine of constant quantities and unchangeable geometric figures.

Basically, it was computational mathematics with its own algorithms for solving arithmetic, algebraic and geometric problems. Negative numbers and their simplest interpretation are introduced. The creation of a decimal positional number system is an outstanding achievement of Indian scientists.

Scientists from the countries of the Near and Middle East took in the "VIII-IX centuries from the Greeks a stock of mathematical knowledge and mastered their deductive method of research. These circumstances allowed the scientists of the countries of Islam to develop computational mathematics much further. They created trigonometry as an extensive big science and separated algebra into an independent discipline. In the formation of the theoretical foundations of computational mathematics, the development of the general theory of relations and the introduction of the concept of an irrational positive number were of great importance.

Mathematicians of medieval Europe during the XIII-XIV centuries not only creatively mastered the mathematical and astronomical knowledge of the countries of the East, but in a number of areas, for example, in algebra, they advanced further. The first ideas of functional dependence, its graphical representation and new infinitesimal techniques were born. They were the first harbingers of the entry of mathematics into a new period of development.

The central core of synthetic practical mathematics of the middle Ages was algorithms for solving standard mathematical problems. The theoretical traditions of the computational and algorithmic direction have strengthened only the algorithmic approach. Types of mathematical problems were grouped around practical problems, which became the structure-forming elements of practical mathematics of the middle Ages. Geometric algebra has given way to the "leader" of computational and arithmetic-algebraic methods.

In the development of mathematics, the introduction of letter symbols was of great importance, with the help of which formulas were introduced. Only after that it became possible to develop general rules for analytical and numerical problem solving and it became possible to cover all mathematical operations with finite quantities.

The introduction of variables by R. Descartes made it possible to study the processes of movement and change. The development of analysis received a powerful impetus in the works of R. Descartes and P. Fermat, who included the entire field of classical geometry in algebra.

The main sources of the development of mathematics of variables in the XVII century were: the demands of rapidly developing natural science and technology; in-depth study of the classics of antiquity - Archimedes and Apollonius. The era of variable mathematics, symbolic algebra, analytical geometry, differential and integral calculus has opened.

The mathematics of Modern times has made a dialectical leap from immutable quantities and fixed relations to mobile changing quantities, the widespread use of limit transitions and functional dependencies. This was accompanied by an increase in the level of accuracy of displaying complex processes and relations of reality in an adequate mathematical form by theoretical mathematics. The problems that have stood since antiquity have found ways to solve them on the basis of new mathematical structures. The creation of mathematical analysis allowed I. Newton to formulate the laws of mechanics and the law of universal gravitation. Mechanics became the basis of the description of natural science. This became possible thanks to the revolution of concepts, which played a decisive role in the formation of theoretical mathematics of Modern times.

In the XVII-XVIII centuries, the Greek ideal of axiomatic construction of mathematics and systematic deduction lost their influence. The mathematics of variables and functional dependence began to develop rapidly. Mathematicians of that time were more eager to obtain analysis algorithms and use it to solve problems of physics and mechanics, although by that time the basic concepts of analysis were not yet clearly defined, but the results obtained satisfied the needs of practice. However, the basic concepts of the new calculus - an infinitesimal quantity and the limit of a variable quantity - remained unclear. The creators of mathematical analysis I. Newton and G. Leibniz could not give a rational explanation for the infinitesimal. Therefore, the new calculation remained unfounded. Mathematicians identified an infinitesimal quantity with a finite quantity, which led to a misunderstanding of the essence of the quantity in general. An infinitesimal quantity was understood then as actually infinitesimal, then simply as zero. The accuracy of the analysis results is explained not by the metaphysical rejection of infinitesimals, but by the efficiency of marginal transitions. An infinitesimal quantity in its quantitative definiteness cannot be either zero or actually infinitesimal, but only potentially infinitesimal. The creators of mathematical analysis could not consciously formulate the concept of a limit, much less put it as the basis of analysis. Hence the misunderstanding of the nature of infinitesimals and the lack of a sufficiently rigorous justification of the analysis. Dalembert came quite close to the concept of the limit, but he did not put it as the basis of the analysis. This task was performed by O. Cauchy in the "Course of Analysis". The use of the theoretical provisions of the new calculus without realizing the limits of their applicability led to incorrect, absurd results.

Almost all major mathematicians of the XVII-XIX centuries took part in the substantiation of mathematical analysis, especially B. Bolzano, O. Cauchy, N.I. Lobachevsky, P. Dirichlet, K. Weierstrass, B. Riemann and others who clarified the concepts of limit transition, continuity, function, differentiation and integration.

In the XIX century, the level of rigor of the proof increased and an "e-5" definition was created instead of understanding continuity as the possibility of decomposing a function into a power series. The idea of potential infinity turned out to be insufficient to fully substantiate mathematical analysis. The research of G. Kantor and B. Bolzano showed that mathematical analysis requires the use of the idea of actual infinity, the concept of an infinite set. To clarify the basic concepts of mathematical analysis, a set of real numbers is taken as its numerical foundation. The set-theoretic approach allowed R. Dedekin-du, G. Kantor, K. Weierstrass to build a theory of real numbers - fundamentals of mathematical analysis. Thus, the emergence of set theory is associated with the needs of analysis and with an in-depth study of the functions of a real variable.

A significant success in the XIX century was the creation of "models" of new concepts in terms of classical mathematics. The first works on the convergence of arithmetic with analysis were aimed at rational numbers. G. Grassman, G. Hankel, K. Weierstrass obtained "models" of positive rational numbers or negative integers in the form of classes of pairs of natural numbers. G. Kantor, R. Dedekind, K. Weierstrass found a "model" of irrational numbers in the theory of rational numbers. In 1861, Grassmann defined addition and multiplication of integers and proved their basic properties: commutativity, associativity, distributability. In 1888, R. Dedekind formulated a system of axioms for arithmetic. The same system of axioms was reproduced in 1891 by J. Peano is also known by his name. The "model" is constructed for complex numbers within the framework of the theory of real numbers. Thus, real numbers are interpreted in terms of integers, complex numbers and Euclidean geometry are interpreted in the same spirit, and the same is true for all new algebraic objects introduced at the beginning of the XIX century by E. Beltrami and F. Klein, who obtained Euclidean "models" of Lobachevsky's non-Euclidean geometries. R. Dedekind showed how the concept of a natural number can be derived from the basic concepts of set theory. In the works of G. Frege, B. Russell, A. Whitehead and D. Hilbert, the formalization of logic is carried out. On this basis, D. Hilbert and his students developed several formal systems of arithmetic.

In order to save set-theoretic mathematics from the paradoxes of set theory and criticism from intuitionists, D. Hilbert and his school undertook a number of new studies in order to substantiate classical mathematics. D. Hilbert called the reasoning methods allowed in metamathematics finite, which avoid the use of actual infinity. In the theory of evidence, the final point of view assumed a concrete and meaningful way of considering and the final attitude of thinking. In order to develop a theory of proofs, D. Hilbert had to carry out the formalization of theories, i.e. replacement of the mathematical theory with the corresponding formal system. D. Hilbert's finitism excludes from use in the theory of proofs those concepts and means that are rejected by intuitionists in classical mathematics.

"Models" based on arithmetic become even more important with the spread of the axiomatic method. Further use of the "model" made it possible in the XIX century to combine mathematics. By the end of the XIX century, the formalization of mathematics ended, and the use of the axiomatic method became a fait accompli. In the XIX century. the use of the axiomatic method in mathematics is required to solve the issues of consistency of mathematical theory. The return to strictness, which arose at the beginning of the XIX century, made some improvement in this issue. Starting with the works of M. Pasha, the rejection of any appeal to intuition becomes a clearly formulated program.

An axiomatically constructed formal theory ceases to be hypothetical only when meaningful interpretations are found for it either in the form of objects of the real world, or in the form of theories that have already found application in practice.

Practice is the starting point of mathematical concepts, but it usually does not act as a direct criterion for the truth of mathematical statements. Only in the end, practice determines the suitability of a particular mathematical apparatus for describing specific phenomena of reality. The criterion of truth in mathematics is the theory of arithmetic of natural numbers, the truths of which are unshakable for every mathematician. Using the arithmetic of natural numbers as a direct criterion of truth means that this criterion is directly related to two other requirements - accuracy in applied mathematics and consistency in theoretical mathematics.

Since the end of the XIX century, the "crisis of foundations" began, which lasted for more than thirty years. The paradoxes of set theory appeared: in the theory of cardinal and ordinal numbers; in the "set of all sets"; in the "set of sets that are not elements of themselves". To exclude the possibility of paradoxes in set theory, formalists sought to betray the axiomatic basis of set theory. E. Zermelo in 1908 gave the first axiomatization in set theory, where he avoided "too large" sets by introducing the "axiom of choice". Then other axiomatic systems appeared. The set theory corresponding to the Zermelo-Frenkel system is used by most mathematicians. Paradoxes in set theory are associated with the use of abstractions that go beyond possible concrete applications, and in this sense it is divorced from the concrete. This means that operating with abstractions cannot be absolutely free. Operating with extremely broad abstractions does not make it possible to point to any specific object associated with this abstraction, or to indicate a possible way to construct it. Set theory has had a fruitful influence on the development of mathematics in general. It led to a significant restructuring of almost all branches of mathematics and helped in the substantiation of modern mathematics.

Having proved the consistency of particular formalized systems covering a part of arithmetic, D. Hilbert and his school believed that they had already achieved their goal and proved the consistency of not only arithmetic, but also the consistency of set theory. A unified mathematical picture of the world - D. Hilbert's attempt to cover all mathematics through axiomatic set theory failed due to the incompleteness of any axiomatic system. According to the incompleteness theorem in a sufficiently developed formal system, it is possible to formulate a sentence that is unprovable in it, as its negation is unprovable in it. Godel's theorem does not speak about the complete collapse of the axiomatic method. Only Hilbert's hopes of formulating such a system of axioms from which the entire true proposition of mathematics and logic could be deduced failed. The results obtained by K.

Goedel pointed to fundamental difficulties on the way of axiomatic construction of mathematical theory, to new limits of applicability of the axiomatic method. Thus, he proved the impossibility of constructing a "universal axiomatic system" not only for the whole of mathematics, but even for its individual sections, for example, arithmetic. Godel's incompleteness theorem has dashed the hopes of mathematicians to make all mathematics formal, but mathematicians themselves do not suffer from this, since the thinking of an applied mathematician is never formal.

During the first quarter of the twentieth century, virtually all mathematics was rebuilt on the basis of set-theoretic concepts. This approach allowed us to establish the logical unity of almost all mathematics. At the end of the thirties of the XX century, French mathematicians under the pseudonym Nicola Bourbaki united in order to build a mathematical picture of the world on an axiomatic basis, where the foundation was set theory. For them, the basic mathematical structures were algebraic, topological, and order structure. Thus, with the help of the most developed mathematical structures, N. Bourbaki sought to embrace with a single eye the fields of mathematics unified by axiomatics. At the same time, the concept of a hierarchy of mathematical structures, going from simple to complex, from general to particular, became the ordering principle. Now particular mathematical theories are losing their former autonomy, becoming objects of study of the most common numerous mathematical structures. This attempt remained incomplete. since the goal itself was apparently unrealistic.

From the point of view of intuitionists, the main thing in mathematics is the method of constructing mathematical objects. That is, objects that cannot be constructed based on a natural series or for the construction of which a method cannot be specified do not have the right to exist in mathematics. Intuitionists replace the actual infinity with a potential infinity, i.e. an incomplete infinity.

The successors of intuitionists - representatives of the constructive direction in mathematics deny the actual infinity, limit the application of the axiom of the excluded third and recognize the existence of only objects actually under construction or an appropriate method can be specified for their construction. Constructive mathematics is based on the abstraction of potential feasibility, which somehow connects the finite with the infinite, excluding the possibility of paradoxes in set theory. From the works of supporters of the constructive direction in mathematics, it became clear that the axiomatic method is not the only method of constructing mathematical knowledge. Representatives of the constructive direction in mathematics recognize only a strict, algorithmically provable concept of existence, which is more acceptable from the point of view of applied mathematics, computational mathematics, mathematical logic, etc.

There are noticeable signs of a departure from the standard, coming from D. Hilbert's understanding of proof as a deductive conclusion. They begin to realize that the proofs are richer, broader in terms of the techniques used than deductive inference and constructive construction. One of the features of modern mathematics is the widespread use of non-constructive reasoning, pure proofs of existence.

In the 20s of the XX century, algebra reached an unprecedented heyday, algebraization of mathematics took place. The modern success of the development of algebra is largely determined by the widespread use of computers and computerization of human activity. All these factors require algorithmization of mathematics. The general scheme of studying many mathematical objects, sometimes very far from algebra, consists in constructing algebraic systems that well reflect the behavior of the studied objects. Algorithmization of knowledge makes it possible to transfer from one area to another not only the results of scientific research, but also methods and techniques for obtaining them in a particular science. Hence, there is one way to algorithmic integration, to the practical orientation of mathematical knowledge, to its arithmetic-algebraic structure, to increase the role of computational methods. All this makes it possible to count numbers as a fundamental mathematical structure that organically combines applied and theoretical mathematics into a single whole. In this regard, the standard of evidence is changing, shifting towards simplicity, clarity, accessibility. Changes in the standards of evidence in modern mathematics are also associated with the penetration of numerical methods into proofs. The penetration of computational methods of proofs into theoretical mathematics and the mass use of computers in applied and computational mathematics lead to the gradual erasure of methodological discrepancies in theoretical and applied mathematics.

Results and Discussion: The unity of theoretical and applied mathematical knowledge is restored in the process of algorithmization and construction of mathematical models, i.e. systematization outside the evidentiary type. The central core of the organization of mathematical knowledge is the construction of mathematical models of specific sciences, connecting together various branches of theoretical mathematics, and orientation to practice is manifested in the form of orientation to other sciences. Now the main "customers" for mathematics are primarily specific sciences. The structural elements of modern mathematics have become the needs of science as a productive force.

Two principles of the mathematical picture of the world: internal principles of mathematics, external principles of mathematics interact with each other, where a mathematical model in applied mathematics acts as an experimental means by which a computational experiment is carried out, and in theoretical mathematics it acts as an object of study.

Conclusion: Thus, the mathematical picture of the world as a form of knowledge representation through mathematical models contributes to the expansion and dissemination of knowledge in all spheres of human activity.

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